An Efficient Inversion Technique for Banded Linear Systems

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Abstract

Using the admittance matrix representation for the description of microwave devices can lead to systems with a banded coefficient matrix. In this paper we describe an efficient procedure for the solution of banded linear systems. A comparative study between the procedure proposed and two other commonly used techniques is performed indicating that the technique presented in this paper is indeed very effective.

I Introduction

The analysis of microwave devices composed of cascaded uniform waveguides generally requires the solution of a linear system which can become the main computational effort [1]. The situation can be improved exploiting the characteristic of the coefficient matrix [2]. Although there are a number of possible methods to perform the analysis of microwave devices of the class discussed in this paper, we find that the admittance matrix representation [3] -[5], is advantageous because it can easily lead to banded linear systems.

In this paper we describe a technique for solving banded linear systems and compare the number of operations required with the ones requred by standard techniques clearly showing the time reduction introduced.

II Basic formulation of the problem

A microwave device consisting of N cascaded waveguides can be described as the connection of N-1 waveguide junctions and N lengths of uniform waveguides. Using admittance matrices, we can obtain a global multimode equivalent network representation (Fig. 1) [5] that, in mathematical terms, is equivalent to a linear system which needs to be solved to find the electrical behavior of the device.

A technique that can be used to solve the system is the 'Gaussian Elimination' technique with 'Backsubstitution'. An alternative procedure is based on recursively reducing two connected admittance matrices to a single matrix until a single Y matrix is obtained [6].

III Reduction to a single Y matrix

This procedure is based on the reduction of two connected multiports (Y matrices) to a single one starting, for instance, from the last two. Proceeding in the same way for the rest of the network, we can finally obtain, after (2N-2) iterations, a single multiport representing the whole structure. Once this is done, to compute the scattering parameters of the microwave device we only need to solve the resulting system. In Table I we summarize the number of operations (multiplications and inversions of complex matrices) that are required as a function of the number of waveguide sections (N).

IV The iterative technique

Considering again the multimode equivalent network representation shown in Fig. 1, we note that a banded linear system can be simply obtained by imposing the continuity conditions for the voltages and currents at the junctions. The next step is then to load the network with the characteristic admittance of each mode at the input and output waveguides, thus obtaining the $2N$ equations

$$I_{uw1} = \left(Y_{uw1}^{(1,1)} + Y_{0uw1}\right) \cdot Z_{uw1}^{(1)} + Y_{uw1}^{(1,2)} \cdot Z_{uw1}^{(2)} \qquad (1)$$

$$O_{uw1} = Y_{uw1}^{(2,1)} \cdot Z_{uw1}^{(1)} + \left(Y_{uw1}^{(2,2)} + Y_{wj1}^{(1,1)}\right) \cdot Z_{uw1}^{(2)} + Y_{wj1}^{(1,2)} \cdot Z_{uw2}^{(1)} \qquad (2)$$

$$O_{uw2} = Y_{wj1}^{(2,1)} \cdot Z_{uw1}^{(2)} + \left(Y_{wj1}^{(2,2)} + Y_{uw2}^{(1,1)}\right) \cdot Z_{uw2}^{(1)} + Y_{uw2}^{(1,2)} \cdot Z_{uw2}^{(2)} \qquad (3)$$

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$$O_{uwi} = Y^{(2,1)}_{uwi} \cdot Z^{(1)}_{uwi} + \left(Y^{(2,2)}_{uwi} + Y^{(1,1)}_{wji}\right) \cdot Z^{(2)}_{uwi} + Y^{(1,2)}_{wji} \cdot Z^{(1)}_{uwi+1} \quad (4)$$

$$O_{uwi+1} = Y^{(2,1)}_{wji} \cdot Z^{(2)}_{uwi} + \left(Y^{(2,2)}_{wji} + Y^{(1,1)}_{uwi+1}\right) \cdot Z^{(1)}_{uwi+1} + Y^{(1,2)}_{uwi+1} \cdot Z^{(2)}_{uwi+1} \quad (5)$$

$$\vdots$$

$$O_{uwN-1} = Y^{(2,1)}_{uwN-1} \cdot Z^{(1)}_{uwN-1} + \left(Y^{(2,2)}_{uwN-1} + Y^{(1,1)}_{wjN-1}\right) \cdot Z^{(2)}_{uwN-1} + Y^{(1,2)}_{wjN-1} \cdot Z^{(1)}_{uwN} \quad (6)$$

$$O_{uwN} = Y^{(2,1)}_{wjN-1} \cdot Z^{(2)}_{uwN-1} + \left(Y^{(2,2)}_{wjN-1} + Y^{(1,1)}_{uwN}\right) \cdot Z^{(1)}_{uwN} + Y^{(1,2)}_{uwN} \cdot Z^{(2)}_{uwN} \quad (7)$$

$$O_{uwN} = Y^{(2,1)}_{uwN} \cdot Z^{(1)}_{uwN} + \left(Y^{(2,2)}_{uwN} + Y_{0uwN}\right) \cdot Z^{(2)}_{uwN} \quad (8)$$

where I_{uw1} is a vector of dimension equal to the number of modes chosen in the first uniform waveguide and $O_{uwi}(i = 1, 2, \ldots, NM(N))$ are null vectors having dimensions equal to the number of modes chosen in the corresponding i-th waveguide. The matrices Y_{0uw1} and Y_{0uwN} are diagonal square matrices containing, respectively, the characteristic admittances $Y_{0i}^{(1)}$ in the elements $(i = 2, 3, \ldots, NM(1))$ of the diagonal, and $Y_{0i}^{(N)}$ in the elements $(i = 1, 2, \ldots, NM(N))$. The vector $Z^{(\gamma)}_{uwi}$ $(\gamma = 1, 2)$ is an unknown vector of *trans-impedances*.

An iterative procedure for the solution of the system can now be developed by starting with equation (8) and writing

$$Z^{(2)}_{uwN} = -Y'_{2N,2N-1} \cdot Z^{(1)}_{uwN} \quad (9)$$

where $Y'_{2N,2N-1}$ is given by,

$$Y'_{2N,2N-1} = \left(Y^{(2,2)}_{uwN} + Y_{0uwN}\right)^{-1} \cdot Y^{(2,1)}_{uwN} \quad (10)$$

Making use of (9) in equation (7) of the system we can now express $Z^{(1)}_{uwN}$ in terms of $Z^{(2)}_{uwN-1}$, obtaining

$$Z^{(1)}_{uwN} = -Y'_{2N-1,2N-2} \cdot Z^{(2)}_{uwN-1} \quad (11)$$

where $Y'_{2N-1,2N-2}$ is given by,

$$Y'_{2N-1,2N-2} = \left(Y^{(2,2)}_{wjN-1} + Y^{(1,1)}_{uwN} - Y^{(1,2)}_{uwN} \cdot Y'_{2N,2N-1}\right)^{-1} \cdot Y^{(2,1)}_{wjN-1} \quad (12)$$

This process can be repeated until we reach the first equation (1) so that the s parameters can be easily computed. The total number of operations required by this method is given again in Table I. As we can see, the iterative technique requires almost half as many multiplications as the reduction technique while both methods require practically the same number of inversions.

V Comparative study

To evaluate the CPU time required for the analysis of a structure, we have used the 4-pole filter shown in Fig. 2 It consists of 29 cascaded sections of waveguides (N = 29). The structure has been analyzed with the multimode admittance matrix approach, the modes of the sections of ridge waveguide and the coupling integrals have been obtained following [9] and [10]. The dimensions of the filter are given in Table II.

Additionally, the Gaussian elimination technique with Backsubstitution has also been implemented. Table III gives the CPU times required to obtain one point in frequency (IBM RISC 6000) with 90 modes in the equivalent network. The iterative technique is clearly the most efficient, reducing the computation time for this example by almost three times.

VI Conclusion

A simple iterative technique has been presented for inverting the banded linear system given by microwave devices composed of sections of uniform waveguides. A comparative study of the CPU time required by other methods has also been performed. The results obtained indicate that the use of the iterative technique can indeed produce substantial reductions in the computational effort.

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Method implemented for solving the linear system	Number of matrix operations	
	Multiplications	Inversions
Reduction technique	6(2N-2)	2N-1
Iterative technique	3(2N-1)	2N

Table I:

Number of matrix operations (multiplications and inversions of complex matrices) required for solving a waveguide structure composed of N waveguides.

Tuning element placed in	Ridge Waveguide			
	a (mm)	b (mm)	w (mm)	h (mm)
1st coupling window	8.707	9.525	2.00	3.444
2nd coupling window	5.107	9.525	2.00	4.071
3rd coupling window	5.109	9.525	2.00	3.564
4th coupling window	5.107	9.525	2.00	4.071
5th coupling window	8.707	9.525	2.00	3.444

Tuning element placed in	Ridge Waveguide			
	a (mm)	b (mm)	w (mm)	h (mm)
1st cavity	19.050	9.525	4.00	3.324
2nd cavity	19.050	9.525	4.00	2.992
3rd cavity	19.050	9.525	4.00	2.992
4th cavity	19.050	9.525	4.00	3.324

Table II:

Dimensions of the 4-pole microwave filter analyzed.

Method implemented for solving the linear system	CPU Time (seconds)
Reduction technique	43.50
Gaussian Elimination technique	23.75
Iterative technique	15.50

Table III:

Total computing time required for inverting at each frequency point the linear system for the 4-pole microwave filter

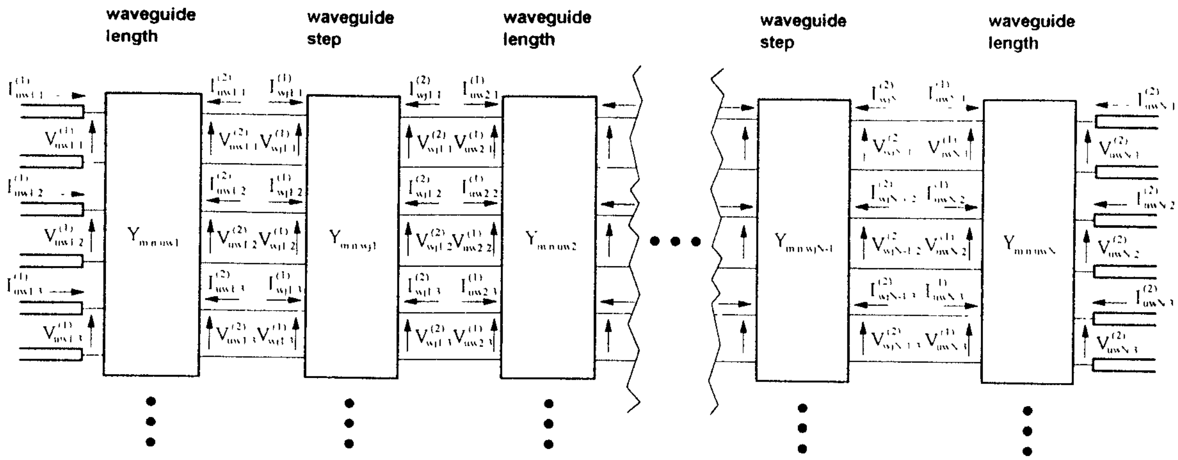

Fig. 1 Global multimode equivalent network representation of a microwave system composed of N waveguides.

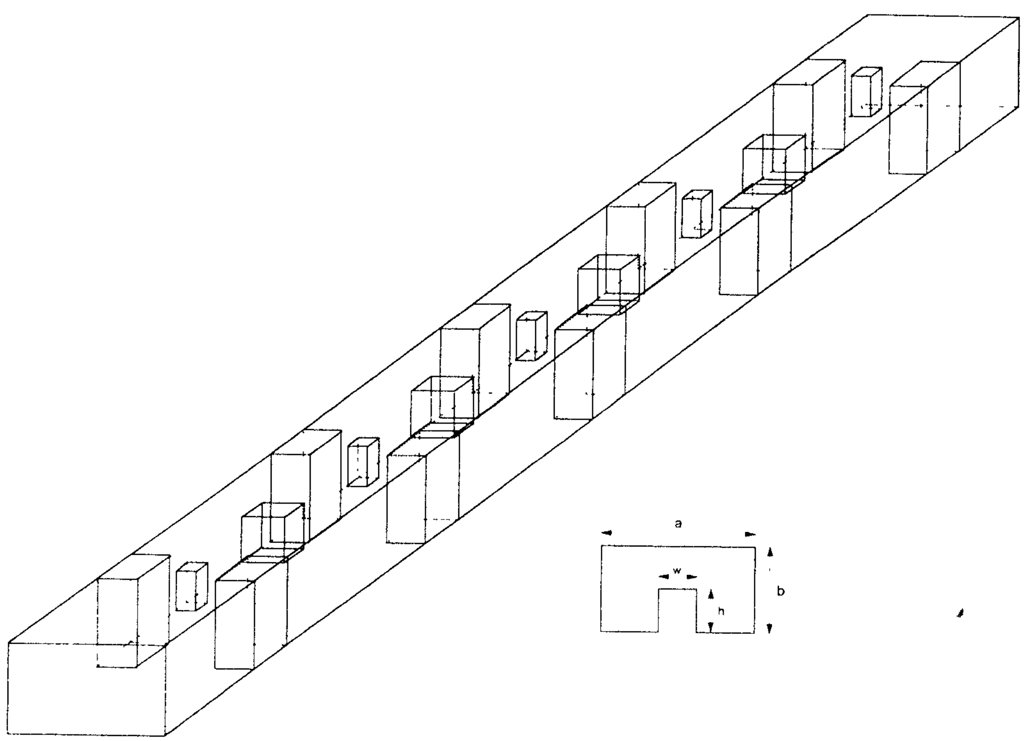

Fig. 2 4-pole microwave filter with tuning elements chosen as an example to evaluate the CPU time required for the analysis of a complex structure (first cavity length 10.5mm, second cavity 13.3mm).

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